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Interim Report to NASA

Mixed Time Integration Methods for Transient*

Thermal Analysis of Structures

by

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Introduction

The purpose of this report is to give a brief summary of the progress made in the study of mixed time integration method for transient thermal analysis of structures during the period from October 1, 1981 to January 15, 1982. A simple and illustrative example problem is also proposed (as suggested by Dr. Olsen) to demonstrate the practicability and usefulness of our proposed approach. We believe that we have now developed an efficient solution procedure for predicting the thermal behavior of aerospace vehicle structures. Currently, a 2D finite element computer program incorporating these methodologies is being implemented. The performance of these mixed time finite element algorithms can then be evaluated employing the proposed example problem. Related discussion is in later section.

Summary of Developments

A family of mixed time integration schemes in which different time integration methods (implicit/explicit) with different time steps can be used in each element group is developed. Its underlying theoretical framework for

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large scale multi-dimensional linear transient analysis is studied. The effective computer implementation aspects of these proposed techniques are also investigated. The "active column equation solver" is the key to success of this technique. The equations system for each element group is constructed in such a way that they are uncoupled and hence each group can be integrated at its own group time step with different integration methods. The stability characteristic of these mixed time partition algorithms is also studied. An energy balance technique is employed to carry out the stability analysis. It is found that the critical time step restrictions are governed by each individual explicit element group only. Of more significance, these mixed time implicit-explicit methods provide a natural framework for the further development of efficient, clean and modularized computer codes.

The above developments are summarized in the attached two papers.

Evaluation of the Mixed Time Implicit-Explicit Algorithms

A number of simple 1D numerical examples are presented in the two papers to demonstrate the feasibilities (i.e. accuracy and stability behavior) of the algorithms. We have formulated herein two finite element thermal models for the Thermal Protection System (TPS) of the space shuttle to evaluate the performance of our mixed time implicit-explicit method. The first one is a 1D model while the second one is a 2D model. They are depicted in figures 1 and 2 respectively. The initial phase of the research project is the development of the mentioned methodologies in which linear assumption is employed. Nonlinear effects such as internal and external radiation, and natural convections are ignored. Further only the assumed mean temperature material properties of the various components of the TPS are considered. Its material properties are tabulated in table 1. The number of elements, the minimum characteristic length L_{min} , the estimated explicit critical time step Δt_{min} ,

the proposed integration method and the proposed element group time step for each group are included in table 2. As can be seen from table 2. due to the various thermal time scales (e.g. $\Delta t_{\min} = 0.083$ for AL, and $\Delta t_{\min} = 133.6$ for RSI), a single integration method is definitely not effective. For example, if an explicit method is employed, a time step of 0.083 has to be used; while if an implicit method is employed, there is no stability-imposed limitation on Δt ; however, wide-banding and/or non-convergence of the resulting matrix equations may decrease its advantage. The family of mixed time integration schemes developed is best suited for this type of problem. The attributes of the various time integration methods are fully achieved using the proposed approach as can readily be seen from table 2. It should also be observed that Δt_{\min} can be set as high as 33.42 in this mixed model. Subsequently $\Delta t = 33.42$ is employed for element groups 1 and 2 (042 Coating and RSI), $\Delta t = 66.84$ for element group 3 (RSI), and $\Delta t = 133.68$ for element groups 4 and 5 (RTV, FELT, AL and AIR). However, judging from the heat loading versus time curve (see figure 3) it is advisable to pick $\Delta t_{\min} = 51.4/4 = 12.85$ during the heat loading period (i.e. time = 0 to 1200) and $\Delta t_{\min} = 33.42$ in later time. Of course, an automatic selection of the step size based on truncation error and accuracy considerations is desirable. It is not being considered here because of the limited time frame and budget of this investigation.

The importance of this proposed mixed time implicit-explicit finite element concept can further be visualized if a nonlinear analysis of the above finite element models is assumed. The thermal responses of the various components of the TPS may be divided into regions of slowly and rapidly varying temperatures and they are:

1. 042 Coating

The thermal diffusivity is almost independent of temperature and the estimate Δt is so small (0.25) that explicit calculation is not cost effective at all. Implicit calculation is best suited since only limited partial reformation and refactorization of this element group's effective stiffness will be needed. This is one of the major advantages of the mixed time implicit-explicit concept.

2. RSI

The thermal diffusivity changes rapidly with temperature, therefore reformation and refactorization of this element group's effective stiffness are frequently required if implicit method is employed. Non-convergence and/or wide-banding of the resulting element group effective stiffness may decrease the advantage of applying the implicit method here. The estimated Δt (based on mean temperature value) is 33.42; therefore explicit is best suited (no matrices operations).

3. RTV

Its thermal diffusivity is independent of temperature and the estimated Δt is too small (0.213) that implicit method is recommended.

4. AL

Its thermal diffusivity is fairly independent of temperature (at least in the operational temperature range of the space shuttle) and its estimated Δt is so stringent (0.083) that implicit method is again recommended.

5. FELT

Its thermal diffusivity is independent of temperature and its

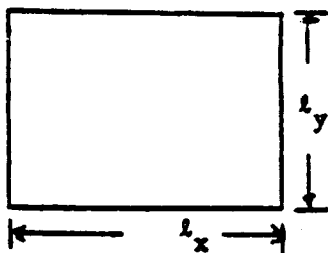
estimated Δt is large (51.4), therefore implicit method or explicit method can be used. For this example, we propose to employ implicit method. However for 3D calculation, in order to reduce the bandwidth and the unnecessary nonlinear calculations (due to the fact that the adjacent groups might be implicit groups too), explicit method will be recommended.

6. AIR

The variations of the thermal diffusivity with temperature are small in our range of interest. Also the estimated Δt is small, therefore implicit method is again well suited.

Remarks:

- (1) In the above Δt calculations, $t_{\min} = \min \{t_x, t_y\}$ where t_x and t_y are defined as follows:



- (2) The estimated Δt is defined to be t_{\min}^2/a . It is a conservative critical time step calculation. Note that a is the thermal diffusivity.
- (3) By virtue of the fact that each element group's effective stiffness is uncoupled to the global assembled matrix equations system, any element group can be reformed and refactorized at any instant if required without affecting the global equations system. This partial factorization procedure can further be enhanced if it is to be combined with an iterative update procedure.

- (4) In the case of short time duration transients, mixed time explicit-explicit is most effective. It is even more attractive if the mixed time explicit-explicit procedures are unconditionally stable.

Conclusions and Future Directions

The work summarized here is not complete. The most important work in progress is the development of mixed time integration methods for transient thermal analysis of structures suitable for incorporation into most finite element computer codes. These mixed time procedures are currently being integrated into a pilot finite element computer program. Detailed studies of solutions of the proposed examples are planned.

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Table 1. Mean Temperature Material Properties

MATERIAL	042 COATING	RSI	RTV	FELT	Al.	AIR
PROPERTIES						
λ w/m-k conductivity	1.4338	0.1354	0.3113	0.0363	131.3573	0.0271
ρ kg/m ³ density	1665.92	144.17	1409.62	96.11	2851.28	1.126
C J/kg-k specific heat	1317.96	1255.20	1213.36	1213.36	962.32	1009.0
$a = \frac{\lambda}{\rho C}$ m ² /sec thermal diffusivity	6.53×10^{-7}	7.48×10^{-7}	1.82×10^{-7}	3.11×10^{-7}	4.78×10^{-5}	2.39×10^{-5}

The mean temperature material properties are computed from the average of those at 1200 k, 950 k, 500 k, 477 k, 333 k and 300 k.

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Table 2. Characteristic Length and Time Scales

ELEMENT GROUP NUMBER	MATERIAL TYPE	NUMBER OF ELEMENTS	$L_{\min}(\text{cms})$	ESTIMATED Δt	INTEGRATION METHOD	ELEMENT GROUP TIME STEP
1	042 coating	20	0.04 ($=\Delta t$)	0.25	I	33.42
2	RSI	50	0.5	33.42	E	33.42
3	RSI	80	1.04	133.6	E	66.84 ($=2\Delta t$)
4	RTV FELT AL	20 10 28	0.02 0.40 0.20	0.213 51.40 0.083	I	133.68 ($=4\Delta t$)
5	AIR	32	1.0	4.18	I	133.68

total number of elements = 240

I = implicit integration

E = explicit integration